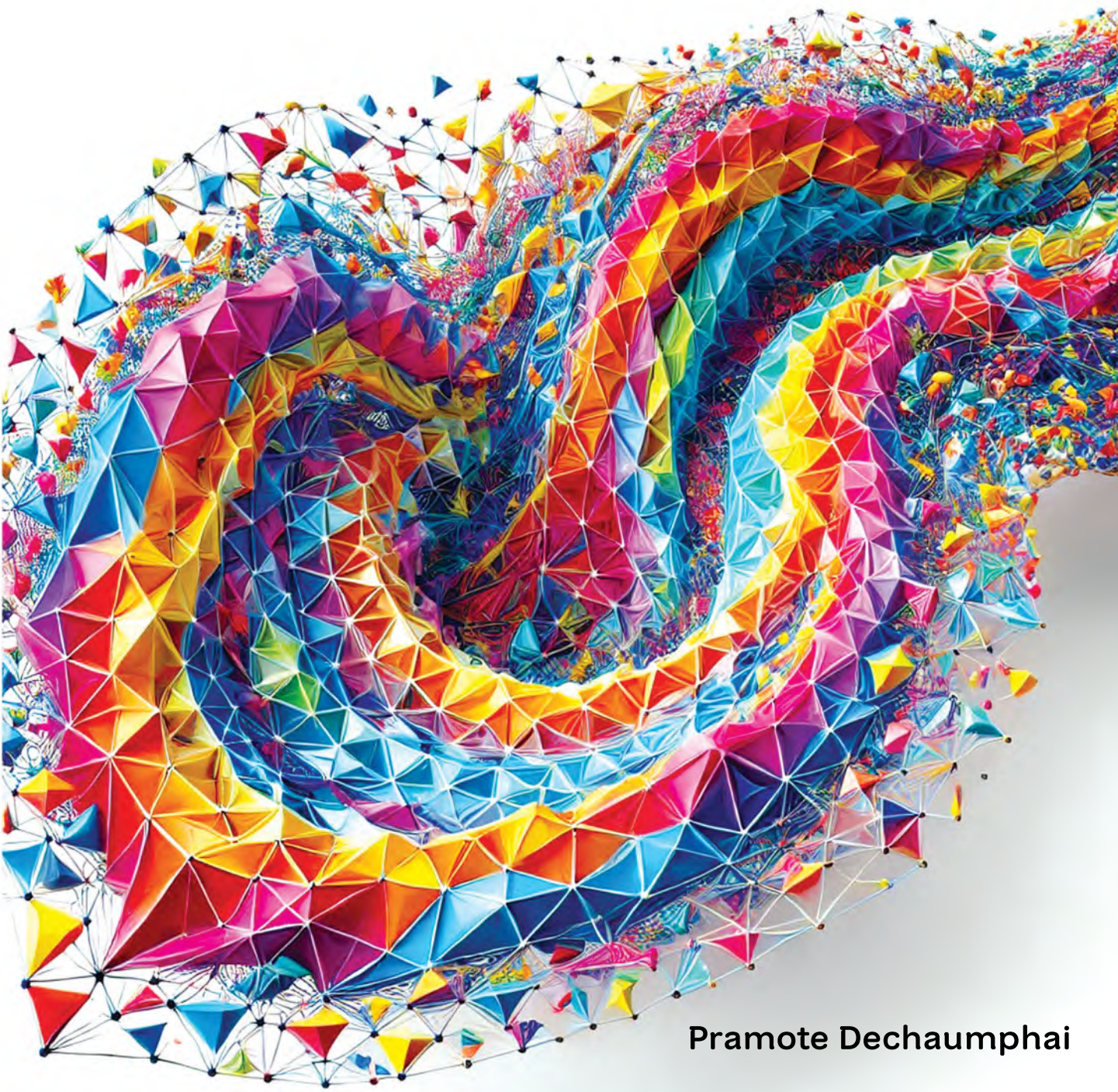
CUpress

Finite Element Handbook



Pramote Dechaumphai

Finite Element Handbook

Pramote Dechaumphai

Finite Element Handbook

Pramote Dechaumphai

Chulalongkorn University press

2026

175.-

Dechaumphai, Pramote

Finite element handbook / Pramote Dechaumphai

1. Finite element method.

620.00151825

ISBN (e-book) 978-974-03-4461-2

CUP 2749

Copyright © 2026 by Chulalongkorn University Press

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, i.e. electronic, mechanical, photocopying recording, etc. Without prior written permission of the publisher.

Published by **Chulalongkorn University Press**

First edition 2026

www.cupress.chula.ac.th [CUB6808-005]

Tel. 0-2218-3562-3

Managing editor: Assoc. Prof. Vimolvan Pimpan, Ph.D.

Academic Editorial Department: Emeritus Prof. Piyanart Bunnag, Ph.D.

Prof. Pramote Dechaumphai, Ph.D.

Prof. Chitsanu Pancharoen, M.D.

Assoc. Prof.Pimpan Dachakupt, Ph.D.

Coordinator: Wasana Sumsen

Proofreader: Bhutsarin Sorrawat

Artwork and Cover: Kaiaumporn Phongkhachorn

Contact: **Chulalongkorn University Book Center**

Phyathai Road, Pathumwan District, Bangkok 10330, Thailand

http://www.chulabook.com

Tel: 08-6323-3703-4

customer@cubook.chula.ac.th, info@cubook.chula.ac.th

Apps: CU-eBook Store

Preface

The Finite Element Method (FEM) software is widely used in both academia and industry nowadays. As users work with these tools to analyze and design scientific and engineering problems, they often start wanting to understand the calculations occurring inside the software. This helps them gain confidence in the accuracy of the results displayed as colorful visualizations on their computer screens.

This "Finite Element Handbook" provides a clear and easy-to-understand overview of the Finite Element Method. It begins with the essential fundamentals of FEM, including the necessary mathematics and numerical methods. The book then explains how FEM works to analyze different types of problems, such as structural analysis, heat transfer, and fluid flow, both steady and unsteady, linear and nonlinear. It also highlights important considerations when using FEM to ensure accurate and reliable results while minimizing computation time.

The author would like to express gratitude to Chulalongkorn University Press for their support in publishing and distributing this handbook in both Thai and English versions.

Pramote Dechaumphai

Contents

Preface

Part I: Understanding the Basics **1**

Chapter 1 Introduction to FEM **3**

1.1 What is FEM? 3

1.2 Real-world Applications of FEM 5

1.3 Why Learn FEM? 6

Chapter 2 Mathematical Foundations Made Simple **9**

2.1 Differential Equations in Engineering 9

2.2 Key Concepts: Nodes, Elements, and Meshes 11

2.3 Basic Linear Algebra for FEM 13

Chapter 3 The Big Picture: How FEM Works **17**

3.1 Breaking Down a Problem into Smaller Parts 17

3.2 Key Steps: Discretization, Assembly, and Solution 20

3.3 The Role of Boundary Conditions 22

Part II: Core Principles Simplified **27**

Chapter 4 The Element Stiffness Matrix **29**

4.1 Derivation Without the Headache 29

4.2 What Does Stiffness Mean? 32

4.3 Understanding Interpolation Functions 34

Chapter 5 Assembly of the Global System **39**

5.1 From Element Matrices to a Global System 39

5.2 Using Symmetry and Boundary Conditions to Simplify Calculations 42

Chapter 6 Solving the FE Equations 47

6.1 Step-by-Step Process 47

6.2 Common Solvers: Direct vs. Iterative Methods 50

Part III: Practical Applications 55

Chapter 7 1D FEM: The Easiest Place to Start 57

7.1 Solving Bar and Truss Problems 57

7.2 Example Walkthrough: Axial Deformation of a Rod 61

Chapter 8 2D FEM Made Easy 65

8.1 Plane Stress and Plane Strain Problems 65

8.2 Examples: Plane Stress and Heat Conduction Problems 67

Chapter 9 3D FEM for Beginners 73

9.1 The Leap to 3D Problems 73

9.2 Tips for Dealing with Complexity 76

Chapter 10 Heat Transfer and Fluid Flow 81

10.1 FEM in Thermal Analysis 81

10.2 Basics of Using FEM for Fluid Problems 84

Part IV: Using FE Software 89

Chapter 11 Introduction to FE Software Tools 91

11.1 Overview of Popular Tools (ANSYS, COMSOL, NASTRAN) 91

11.2 Advantages of Learning Software Early 93

Chapter 12 A Beginner's Guide to FE Software **97**

12.1 How to Set Up a Simple Problem 97

12.2 Interpreting the Results: What Do the Colors Mean? 99

Part V: Advanced Topics for the Curious **103**

Chapter 13 Dynamic Problems in FEM **105**

13.1 Basics of Time-dependent Analysis 105

13.2 Solving Vibration Problems 108

Chapter 14 Nonlinear Problems Simplified **113**

14.1 What Makes a Problem Nonlinear? 113

14.2 Strategies to Approach Nonlinearities 117

Chapter 15 Tips for Mesh Optimization **121**

15.1 Choosing the Right Element Type and Size 121

15.2 Avoiding Common Pitfalls in Meshing 124

Part VI: Final Words **129**

Chapter 16 Common Mistakes and How to Avoid Them **131**

16.1 Troubleshooting in FE Model 131

16.2 Learning from Errors 134

Chapter 17 Where to Go Next **139**

17.1 Resources for Deeper Learning 139

17.2 Applying FEM in Our Field 143

Chapter 18 Glossary, Formulas, Software Books **147**

18.1 Glossary of FEM Terms 147

18.2 Useful Formulas and Tables 151

18.3 FE Software Books 153

Part I

Understanding the Basics

Chapter 1

Introduction to FEM

1.1 What is FEM?

The Finite Element Method (FEM) is a powerful numerical technique used to solve complex problems in engineering and science. It provides a way to model and analyze systems that are too complicated to solve with traditional analytical methods.

1.1.1 Key Concepts

FEM is a numerical method that transforms a large, complicated problem into smaller, simpler pieces called *finite elements*. These elements are connected at points called *nodes*. FEM is primarily used to approximate solutions to problems involving:

- Structural mechanics
- Heat transfer
- Fluid dynamics
- Electromagnetic fields
- Etc.

1.1.2 How FEM Works

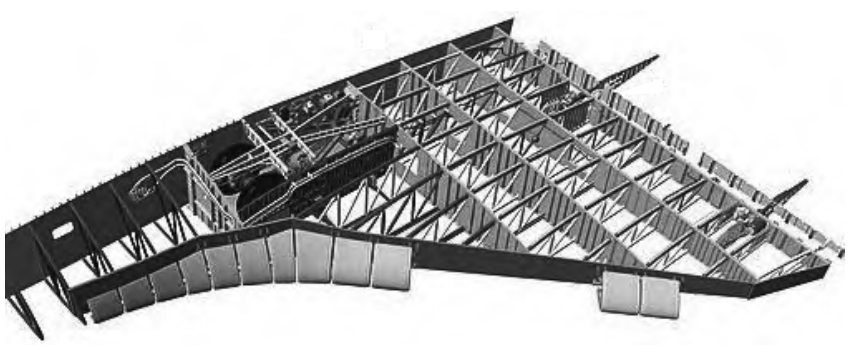
a) **Discretization:** The system or structure is divided into a mesh of smaller elements, allowing complex geometries and behaviors to be represented more easily.
b) **Mathematical Modeling:** Each element is described by mathematical equations based on physics or engineering principles (e.g., equilibrium, conservation laws).
c) **Assembly:** The individual equations for all elements are assembled into a larger system of equations representing the entire model.
d) **Solution:** The resulting system of equations is solved, often using computers, to determine values like displacement, temperature, or pressure at various points.

1.1.3 Why FEM is Important

a) **Versatility:** FEM can handle complex geometries, material properties, and boundary conditions that are difficult to analyze otherwise.
b) **Accuracy:** By refining the mesh (increasing the number of elements), solutions become more accurate.
c) **Wide Applicability:** FEM is used in various fields such as aerospace, automotive, civil engineering, biomedical engineering, and energy.

1.1.4 A Real-Life Example

One example of FEM in action is its use in designing the space shuttle wing. Engineers simulate stresses and strains on the wings under different flight conditions to ensure safety and performance.



1.2 Real-world Applications of FEM

FEM is a versatile tool widely used in engineering and science to solve real-world problems. Its ability to simulate physical phenomena with precision has made it indispensable across various industries.

1.2.1 Applications by Field

a) **Aerospace Engineering**
- **Aircraft Design**: FEM is used to analyze the stresses and deformations in aircraft wings, fuselages, and landing gears.
- **Space Exploration**: Structural analysis of satellites and spacecraft under extreme conditions like vacuum and high temperatures.
- **Vibration Analysis**: Ensuring aircraft safety by studying natural frequencies and mode shapes of components.

b) **Automotive Industry**
- **Crash Testing**: Simulating car crashes to improve passenger safety and optimize energy absorption.
- **Thermal Analysis**: Studying heat dissipation in engines, brakes, and exhaust systems.
- **Design Optimization**: Developing lightweight yet durable car frames and components to enhance performance.

c) **Civil Engineering**
- **Building Design**: FEM helps in evaluating structural integrity under loads like wind, earthquakes, and snow.
- **Bridge Engineering**: Simulating stress and strain distributions in bridges to ensure safety and durability.
- **Geotechnical Analysis**: Modeling soil-structure interaction to assess foundation stability.

d) **Biomedical Engineering**
- **Prosthetics and Implants**: Designing and testing artificial joints, dental implants, and other prosthetics for performance and durability.
- **Tissue Engineering**: Simulating mechanical behavior of soft tissues under different conditions.
- **Medical Devices**: Optimizing the design of stents, catheters, and surgical tools.

e) **Energy Sector**
- **Wind Turbines**: Evaluating blade performance under aerodynamic and structural loads.